Compressed Sensing Microscopy with Scanning Line Probes

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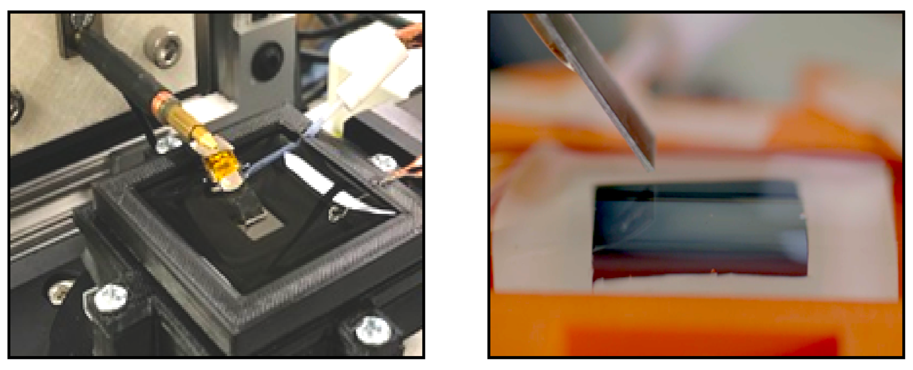
Microscope with Line Probe

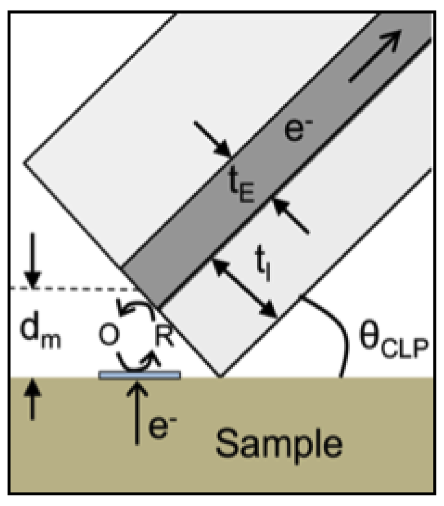
- Conventional scanning probe microscope takes *point measurements*; inefficient sampling.



Line measurements can improve efficiency by order of magnitude on structured signals.

Electrochemical Line Probe

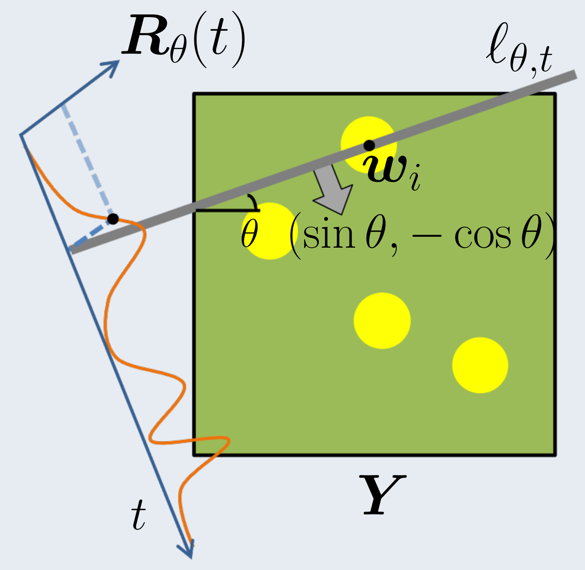
- Line probe measures redox reaction (O $\leftrightarrow$ R) electric current (e^-) induced by conducting layer and electroactive species on the sample.
- Insulating layers sandwich conducting layer w/one edge contacting sample at tilt angle θ_{CLP}. Distance of conducting layer to sample is d_m [1] .



Line Scan Math Model

- *Line projection*: integrate current over line $\ell_{\theta,t}$

$$\mathcal{L}_\theta[\boldsymbol{Y}](t) := \int_{\ell_{\theta,t}} \boldsymbol{Y}(\boldsymbol{w})\,d\boldsymbol{w}$$
$$:= \int_s \boldsymbol{Y}\left(s \cdot \boldsymbol{u}_\theta + t \cdot \boldsymbol{u}_\theta^\perp\right) ds.$$

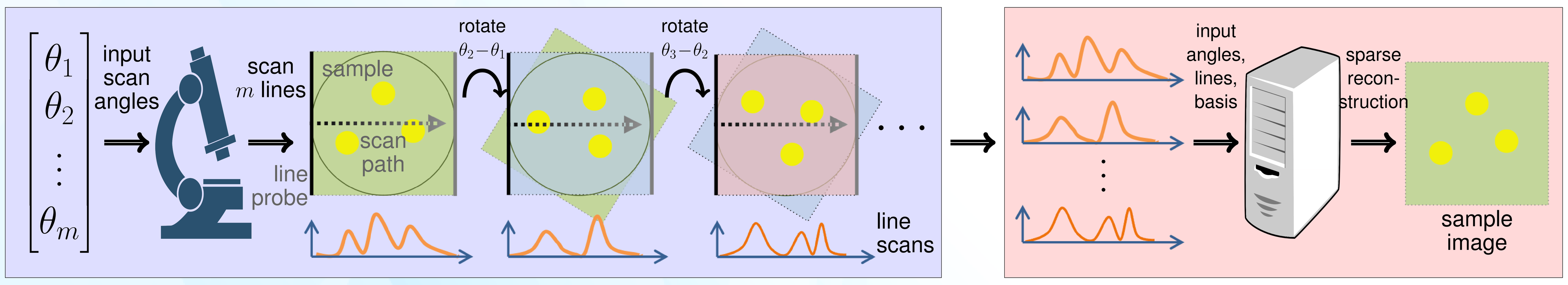


- *Line scans*: sweep line probe m times at different angle with PSF $\boldsymbol{\psi}$ along scan path

$$\widetilde{\boldsymbol{R}} = \tfrac{1}{\sqrt{m}}[\boldsymbol{\psi} * \mathcal{L}_{\theta_1}[\boldsymbol{Y}], \dots, \boldsymbol{\psi} * \mathcal{L}_{\theta_m}[\boldsymbol{Y}]]$$
$$:= \boldsymbol{\psi} * \mathcal{L}_\Theta[\boldsymbol{Y}]$$

take equispaced discrete samples $\boldsymbol{R} = \mathcal{S}[\widetilde{\boldsymbol{R}}]$.

Schematic of Microscopic Line Scans



Compressed Sensing with Line Scans

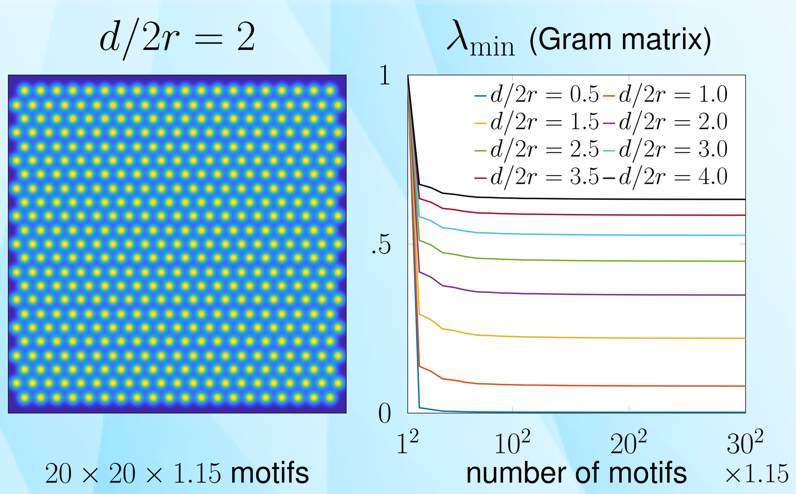
Line scan is **delocalized**, more efficient when sample **spatially sparse** image. We study using the line probe to measure image consists of multiple electroactive discs with small known radius (spatially sparse), it is more efficient than point probe with raster scan.

Case 1: Highly small and separated discs

- Let image has k discs radius r with centers separated by at least $\frac{2}{C}k^2 r$, then three iid uniform random line scans recover the image w.p. at least $1 - C$.

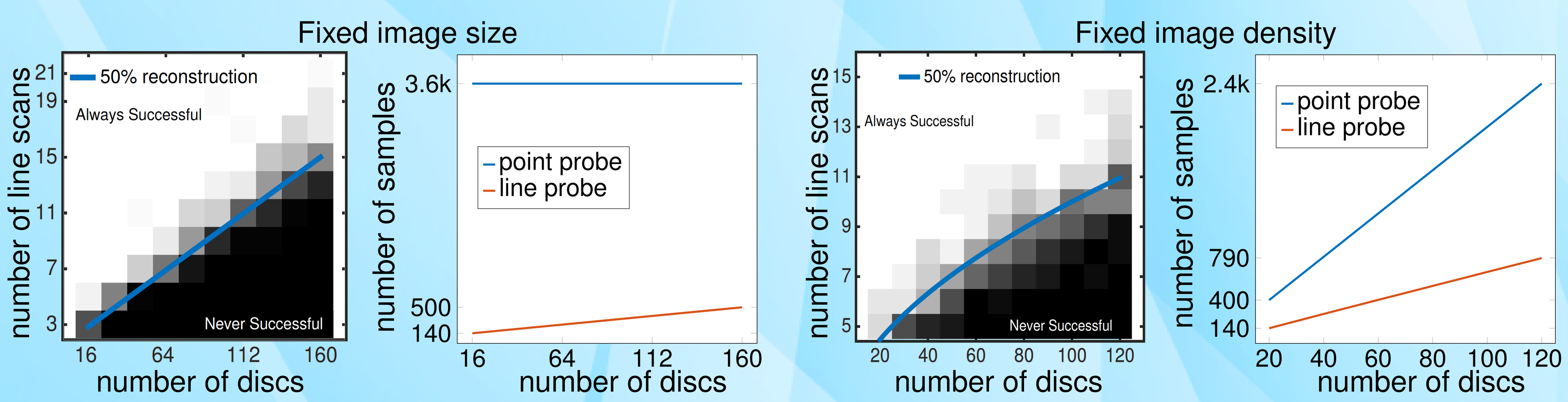
Case 2: Stable injectivity with infinite line projections

- Infinitely many line projection is partially *coherent* with discs $\boldsymbol{D}$ (distance d, radius r):
$$\mathbb{E}_\Theta \left\langle \mathcal{L}_\Theta[\boldsymbol{D} * \boldsymbol{\delta}_{\boldsymbol{w}_i}], \mathcal{L}_\Theta[\boldsymbol{D} * \boldsymbol{\delta}_{\boldsymbol{w}_j}] \right\rangle \approx \tfrac{1}{\sqrt{1+d^2/4r^2}}.$$
not conventional ideal CS measurement.
- When the discs are **separated** ($d/2r > 1$), then $\mathbb{E}_\Theta \mathcal{L}_\Theta[\boldsymbol{D} * \cdot]$ is stably injective over the sparse support, regardless of support number.
- Infinite line projections $\mathbb{E}_\Theta \mathcal{L}_\Theta$ is *lowpass*, can recover discs with enough separation [2].



Case 3: Sparse recovery with finite line projections

- When discs are $d > 2r$ **separated**, the number of line scans required for exact recovery is about **linear proportional** to the number of discs.



Finding: Sample Complexity of Line Scans

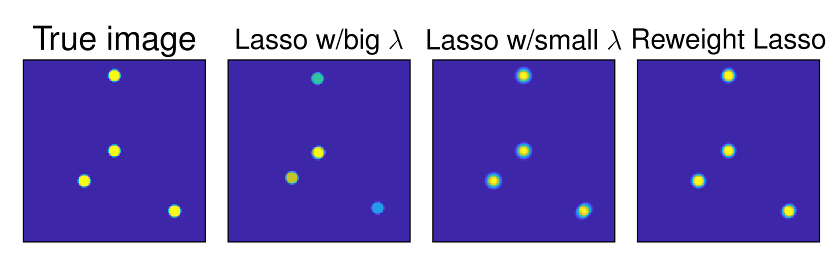
When local features are well separated, the number of line scans required for exact recovery is about linear proportional to number of discs.

Image Reconstruction from a few Line Scans

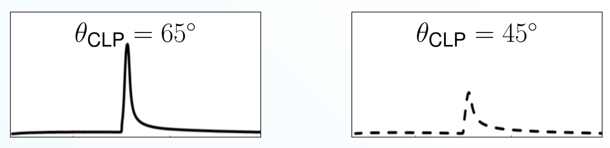
Practical reconstruction from line scans poses additional difficulties: the real measurements are partially **coherent** and exhibit **nonidealities**.

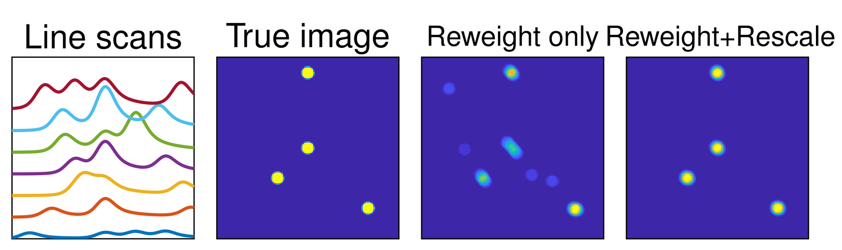
Problems of Vanilla Lasso for Reconstruction

- Incorrect scale recovery: Lasso solution of high coherence $\boldsymbol{A}$ on support Ω:
$$\boldsymbol{X}_{ij} = \left[\boldsymbol{X}_{0ij} - \lambda(\boldsymbol{A}_\Omega^* \boldsymbol{A}_\Omega)^{-1} \boldsymbol{1}\right]_+ \quad \boldsymbol{w}_{ij} \in \Omega$$
has wrong (relative) scale since $\boldsymbol{A}_\Omega^* \boldsymbol{A}_\Omega \not\approx \boldsymbol{I}$.



- Uncertain PSF: Due to physical limitation the PSF varies between samples





Algorithm: Reweighting Calibrated Lasso

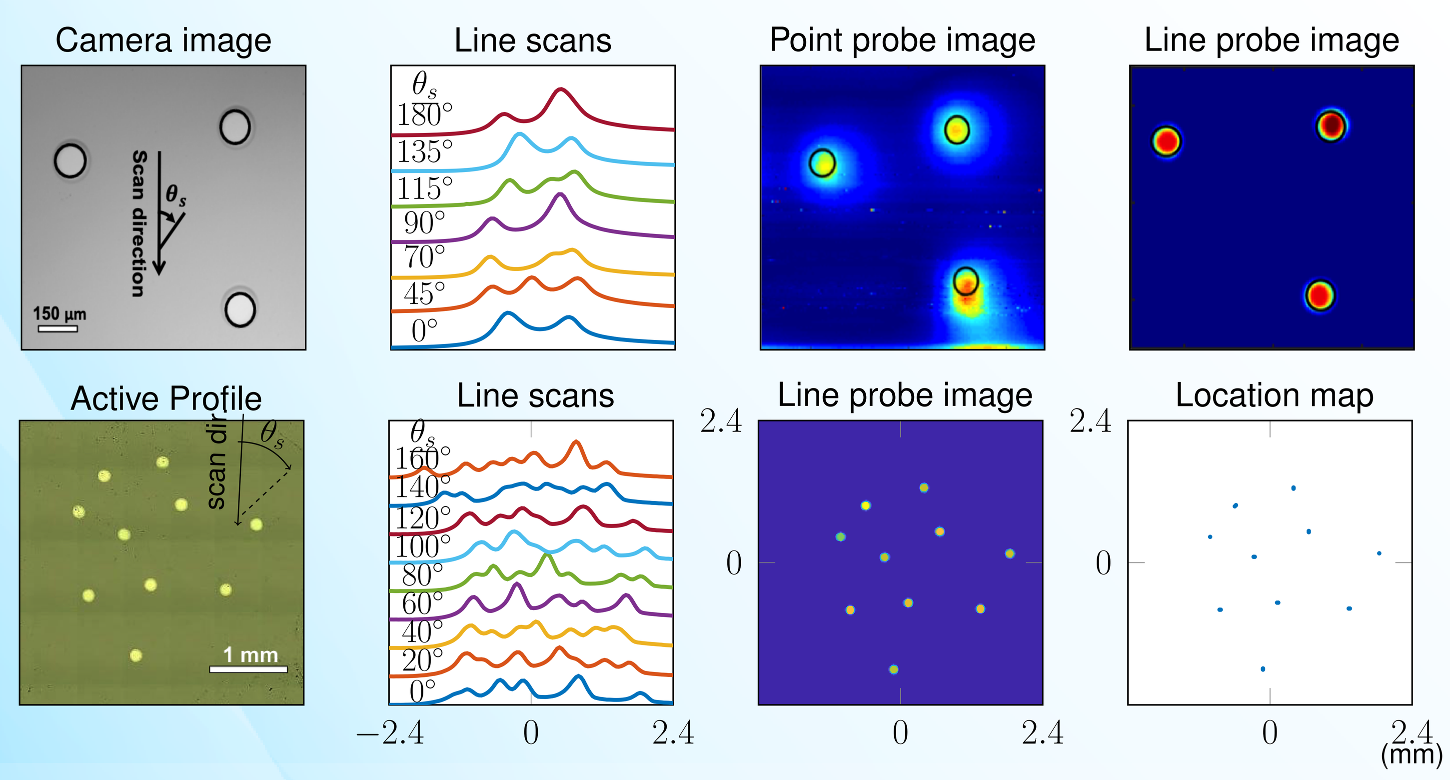
We solve minimization for location map $\boldsymbol{X}$ *and PSF parameters* $\boldsymbol{p}$

$$\min_{\boldsymbol{X} \geq 0, \boldsymbol{p} \in \mathcal{P}} \sum_{ij} \boldsymbol{\lambda}_{ij}^{(k)} \boldsymbol{X}_{ij} + \sum_{i=1}^m \tfrac{1}{2} \left\| \mathcal{S}\{\boldsymbol{\psi}(\boldsymbol{p}_i) * \mathcal{L}_{\theta_i}[\boldsymbol{D} * \boldsymbol{X}]\} - \boldsymbol{R}_i \right\|_2^2$$

with reweighed sparse penalty $\boldsymbol{\lambda}_{ij}^{(k)} = C/(\boldsymbol{X}_{ij} + \varepsilon)$.

Real Data Experiments

- We demonstrate reconstruction from line scan on 3, 10 Pt discs samples.



References

[1] O'Neil, G. D., Kuo H. W., Lomax, D. N., Wright, J. and Esposito, D. V., "Scanning Line Probe Microscopy: Beyond the Point Probe.", Analytical chemistry 90.9 (2018).

[2] Candès, E. J. and Fernandez-Granda, C., "Toward a mathematical theory of super-resolution", *Comm. on pure and applied Mathematics* 67.6 (2014).